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1 MACROMATERIALS

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Structural form is defined in this paper as any periodic shape, and any form (perforated, corrugated, tubular or latticed, for example) is shown to generate its own material from its parent material, just as foam is the vehicle for transforming a material like plastic into another material, foam plastic. It is shown that there are material properties existing at macroscopic scales which are relevant to structural performance, safety and economical design which as yet are not given the recognition we believe they deserve.

INTRODUCTION

It is well known that material strength is a property that is influenced by scale: tiny fibres of glass are much stronger per unit area than rods made of identical material. This can be explained by the defect patterns within the glass. What we suggest in this paper is novel: it is the more general idea that wherever there are patterns not only of defects but also of grains and, at the macroscopic scales, structural forms such as corrugation, perforation, tubularity and latticing, there is a different set of material properties for each scale and the sets associated with the macroscopic scales are particularly deserving of attention.

SHAPE AND FORM

Any shape can be modelled mathematically by a presence field, $s(x,y,z)$, which can take the value 1 or 0 depending on whether the point (x,y,z) is inside or outside the shape. Figure 1 shows a two dimensional shape: the points where $s = 1$ are covered by dark shading, while those points where $s = 0$ are covered by light shading. This shape, like every other, has a closed boundary corresponding to the black line on the figure. $s(x,y,z)$ may be defined by a common scalar function $f(x,y,z)$ by taking s as 1 wherever f is positive and 0 elsewhere. Shapes defined this way are not necessarily as simple topologically as the one illustrated as they may consist of unattached pieces and there may be holes, corresponding to islands and lakes of a typical land mass. The function f could correspond to height of ground above sea level and the boundary, i.e. the coastline, would be defined by $f(x,y) = 0$.

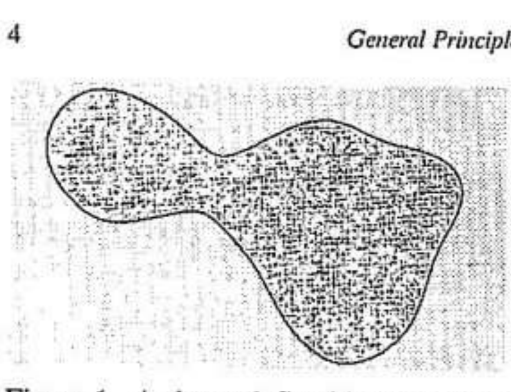

Figure 1. A shape defined by a boolean presence field, $s(x,y)$ taking values 1 and 0 inside and outside.

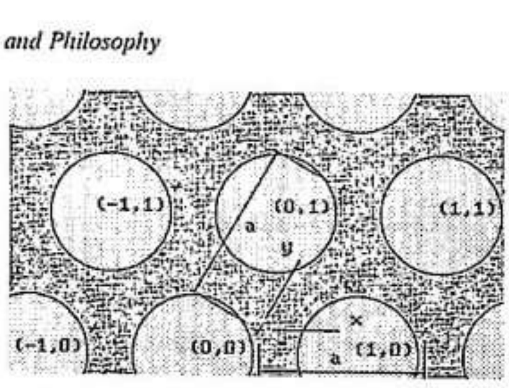

Figure 2. A periodic shape, of form, defined by a periodic presence, ~~values~~ field $s(x,y)$

When the function f is periodic in any or all of x,y or z the shape s that is generated by f is repeating, and we shall call any shape with this property a form. Figure 2 shows a perforated form, a shape which is repeated in two directions with a pitch of a. Pitch can range from the fraction of a nanometre of atomic lattices to the 20 m. member spacing of an offshore structure. The 'address' of any point within this form is its coordinates (x,y), only the periodicity of the form makes it appropriate to separate these coordinates into components (i,j) and (x',y') where (i,j) specifies the cell and (x',y') specifies the location within the cell. If the pitches in the x and y directions are a and b, then

$$x = ai + x'$$

and

$$y = bj + y'$$

Axes are not necessarily orthogonal: they are at 60 deg. to each other in the example of Figure 2 i and j must be integers and x' and y' must always lie between zero and a and b respectively. Varying i and j while keeping x' and y' fixed gives a set of *lattice points*. When a and b are both equal to 1, i and j correspond to the integer parts of x and y to the left of their decimal points, and x' and y' correspond to the non-integer parts. In other words form has just the same hierarchical nature as the arabic numbering system, which is able to describe any number using just ten digits: in a four digit number like the date 1066 the leftmost digit defines the millennium cell, the next the century cell and so on. Another sign of the hierarchical nature of forms is that there are two sorts of adjacent point in the x direction, x + dx and $x + a$, one on the infinitesimal scale and the other, the adjacent lattice point, on a finite scale. Traditional continuum mechanics is founded upon the infinitesimal scale. With forms, finite scales and finite differences assume a new significance.

MATERIAL

A shape is given physical reality when it has material inside it, preferably material with noticeable properties. Less noticeable material. like air, is ideal for the outside if the shape is to be noticed. Mathematically, material is a property field, having values for properties like density, resistivity and stiffness at every point. For example suppose we have material of density $m(x,y)$ shown in Figure 3. Filling the shape $s(x,y)$ of Figure 1 with the material is the multiplication of $s(x,y)$ and $m(x,y)$ to give the structure shown in Figure 4. This confirms the statement made by Parkhouse[1] that

$$\text{shape} \times \text{material} = \text{a structure.}$$

Another assertion made in that paper was that

$$\text{form} \times \text{material} = \text{another material.}$$

When material is dispersed in a repeating pattern its presence is partial: a proportion of any volume considered is occupied by material, and provided the sample volume is chosen carefully this proportion has a value unique to the form. Volumes (or areas in 2-D) which satisfy this sample criterion are called cells, and two such 2-D cells are shown in Figure 5. In 2-D the boundary of one of these cells must be made up of four lines joining any set of adjacent lattice points and opposite lines should be identical to each other apart from a translation by a pitch length in the direction of the other axis. It is clear from the figure that every feature of the form appears exactly once in every unit cell, the complete boundary of one hole for example. We have defined sparsity, i, as the reciprocal of the partial presence factor, i.e.

$$i = \frac{\int_C \mathrm{d}A}{\int_C s\,\mathrm{d}A}$$

where C describes integration over a cell. In 3-D the integrations would be over volume. The sparsity, i, which Parkhouse has also called the dilution factor[2], is a

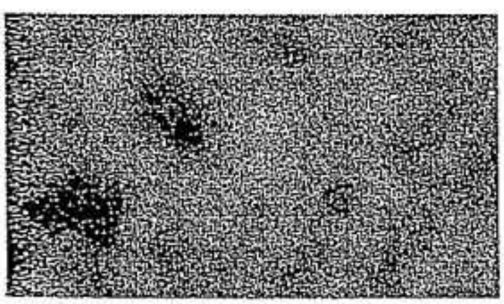

Figure 3. A material of varying intensity $m(x,y)$.

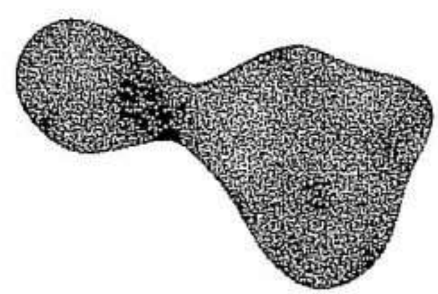

Figure 4. A structure composed of the shape of Fig.1 and the material of Fig.3.

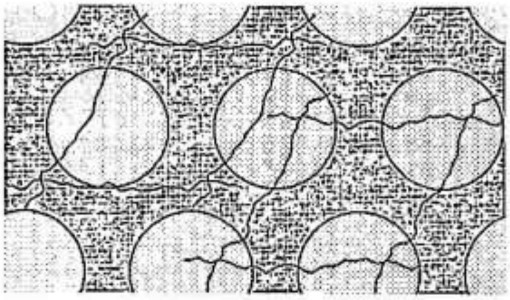

Figure 5. Two unit cells. Each contains just one of each feature of the form.

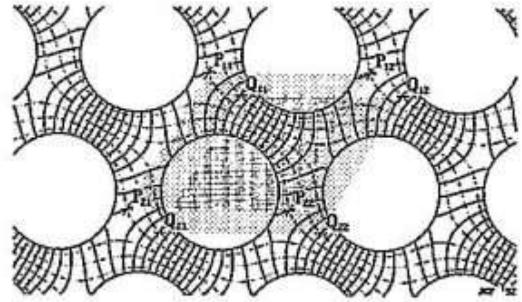

Figure 6. Equipotential solid lines and dotted flow lines through the form.

measure of how sparsely material is spread. It is a property which is independent of which particular point (x,y,z) within the form is being considered so it may be attributed to a uniform material solidly filling the whole volume. Suppose the perforated form is filled with a material of density ρ. Because of the voids in the form the average density will be ρ/i and this average density may be attributed to a uniform material solidly filling the whole area. The material of density ρ will be referred to as the *parent* material and the material of density $\rho' = \rho/i$ as the *equivalent* material. The parent material has been transformed by the form into a different material. Simply, **shape is a transformer of material.**

MATERIAL PROPERTIES

Where there is no density we conventionally agree that there is no material, which is why density may be regarded the most important material property. There are other material properties as simple as density: temperature and electric potential, for example. From these come more complicated properties like potential gradient, current density and resistivity. Resistivity, like density, is an important material property because of its constance: it is relatively unaffected by its environment and distinguishes materials from each other. This cannot be said of temperature. Figure 6 shows a flow pattern in a perforated plate: the dotted lines are currant flow lines and the solid lines are lines of equipotential. Resistivity has three components r_{xx}, r_{yy} and r_{xy} which may be estimated for the equivalent material from measurements of potential difference across adjacent lattice points and the summation of total current flow between adjacent lattice points. A close look at Figure 6 can reveal that both cells P and Q give exactly the same measurements for potential drops and summed flows, demonstrating again that the resistivities of the equivalent material solidly filling the plane are continuum properties of the same nature, but not the same values, as the resistivities of the parent material.

MACROSTRAIN

It should not now come as a surprise to discover that a new set of stresses, strains and stiffnesses arise as a consequence of a form. Figure 7 shows a latticed form deforming from one state defined by a,b and θ to another defined by a',b' and θ'. Note that the deformation pattern is exactly the same in each of the cases (b) and (d): the only difference is that different points on the same form have been chosen as reference points, and the point of the two examples is that the deformation states measured between adjacent lattice points are identical whichever reference point is chosen. The values of a,b and θ are precisely the same in both (a) and (c), and so are the values of a',b' and θ' in (b) and (d). The only difference between (b) and (d) is the rigid body rotations of the two lattices caused by the zero rotation imposed at the reference point. When θ is 90° and the strain is small, the deformation shown by the figures would correspond to engineering strains

$$e_{11}=\frac{a'-a}{a} \qquad e_{22}=\frac{b'-b}{b} \qquad \gamma_{12}=\theta-\theta'$$

These equivalent material strains are an entirely different set of strains from the parent material strains which also exist at every point. Since such strains in

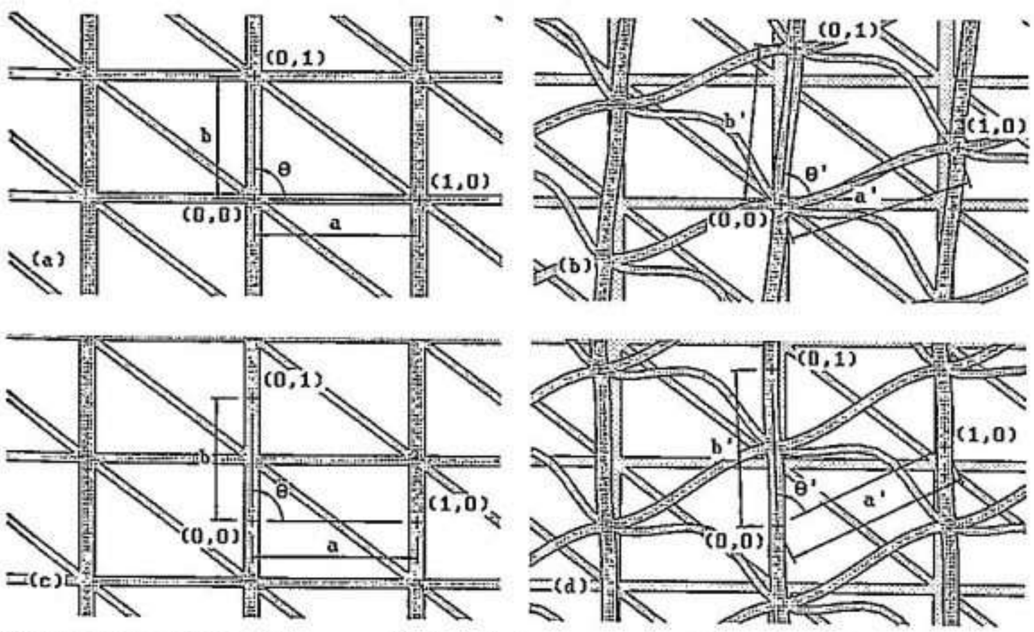

Figure 7. The latticed form on the left is shown on the right after it has undergone a deformation. The difference between the upper and lower illustrations is only the difference in choice of reference point, and this figure illustrates that whatever point is chosen for this purpose the measures of the undeformed state (a,b,θ) and the deformed state (a',b',θ') are the same.

engineering structures occur at rather large scales we shall refer to them as macrostrains. The terms macrostress and macrostiffness will be coined for the same purpose.

What we have shown is that material properties exist at scales other than the infinitesimal. The significance of this to engineering design and development would be difficult to over-estimate, but just now we must admit there are still theoretical difficulties to overcome: when a property like macrostrain is not uniform, the value at the point x could be defined in 1-D as that obtained by averaging either between x and $x + a$, or between $x - a/2$ and $x + a/2$, or between $x - a$ and x. This may be a trivial problem, simply requiring an arbitrary choice, but consider the difficulties of defining macroproperties where the form itself is not uniform, but has varying pitch, and worst of all where a form is discontinuous, as it is at its boundaries: such discontinuities of form are the rule rather than the exception in engineering structures so the measurement or computation of macroproperties over the most commonly encountered structures is as yet problematic.

A form that is periodic in one direction only is relatively free of these problems, its boundaries being confined to its ends. Figure 8 shows a beam, (a) in its undeformed state and (b) deformed. Its deformed state can be expressed in terms of the deformation of its neutral axis by the macrostrains ϵ, γ and κ. ϵ is the axial strain $(a' - a)/a$, γ is a shear strain equal to the angle indicated in the figure and $\kappa = 1/R$ is the curvature of the neutral axis. Which point on the neutral axis is chosen as reference point is again not going to affect the values of the macrostrains measured. What macrostresses, derived from axial force, shear force and bending moment, might be associated with these macrostrains, and could they be matched by a macromaterial stiffness? For incremental strains it turns out that there is always a *unique solid section*, which we call the *equivalent section*, which for a 3-D member is elliptical with its centroid at the neutral axis, which when filled with a *unique uniform material*, which we call the *equivalent material*, behaves for small increments identically to the original beam configuration. This requires the equivalent material to posses appropriate values of Young modulus, two shear moduli and a torsional modulus.

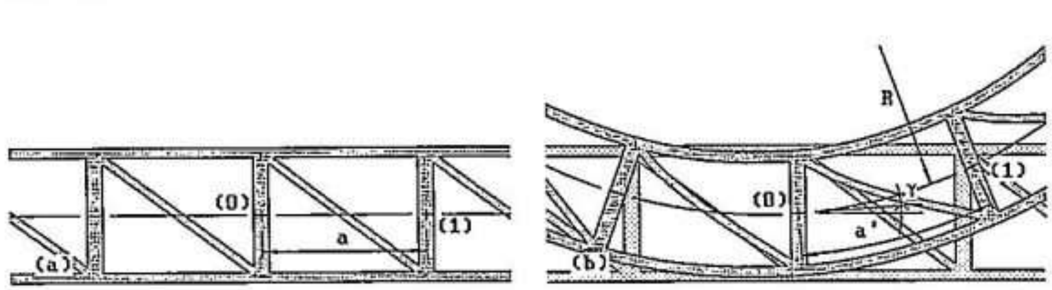

Figure 8. The deformed state of a form periodic in one direction only can be described by the axial strain $\epsilon = (a' - a)/a$, the shearing angle γ and the curvature $\kappa = 1/R$.

SPARSITY

Figure 9 shows three common engineering sections filled with the darker shaded parent material. When this parent material is of uniform stiffness, E, the equivalent solid sections are those illustrated behind, and these are shown filled with uniform equivalent material having such stiffness, E', that the axial and flexural stiffnesses of the equivalent construction are identical to those of the original sections of parent material. When the parent material is uniform it can be shown that the area of the equivalent section, A', must always be greater than or equal to the area of the original section, A. In the first two examples of Figure 9 the ratio A'/A is exactly 10, and we call this ratio sparsity: this definition for shapes is very similar in character to the earlier definition for forms. The elliptical section is that which has the same radii of gyration as the original one. Since both sections with their proper materials are identically stiff it must follow that E/E' must also be exactly 10, and the effect of both the I-section and the tube is to synthesize a material which is 10 times less stiff than its parent material. The densities of the two materials are also in this ratio. This is another illustration that **shape is a transformer of material**, and it also demonstrates that concave shapes like the I-section and the tube introduce sparsity by dispersing material away from itself. In this respect **structuring is a process of material dilution**[2]: latticing, stiffening and tubularity are all ways of making a little material go a long way. A steel I-section is structurally identical to a solid timber section: they serve the same purposes and it is no coincidence that the ratio of both stiffnesses and densities of steel/timber is about 10. Even a solid rectangular section like that shown to the right of Figure 9 has an equivalent elliptical section of greater area, which is why the solid ellipse is chosen: it is the least structured shape, always having a sparsity of one. A rectangle has a sparsity of $\surd(\pi/3)$.

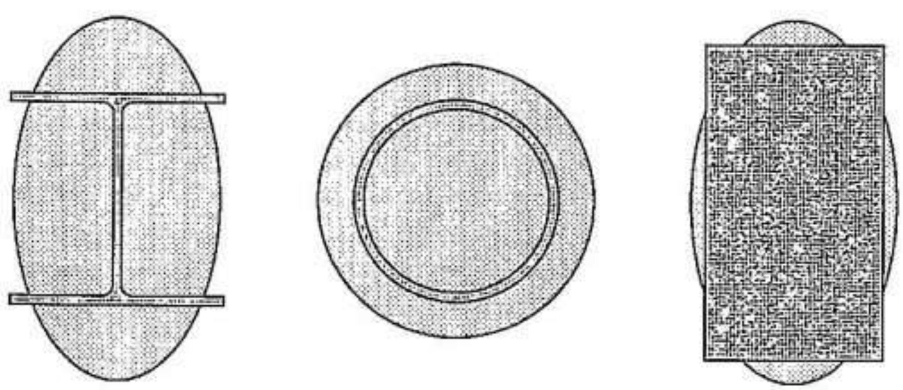

Figure 9. Three common sections in front of their 'equivalent' solid elliptical sections.

APPLICATION TO STRUCTURAL TESTING AND DESIGN

Hooke's Law, discovered in the mid 1600s, was of limited use in structural design until Cauchy expressed the *material* concepts of stress and strain in the mid 1800s when Hooke's Law was expressed as E = stress/strain, a formula that now pervades every structural calculation. We have yet to embrace this material reinterpretation at macroscales. Instead of leaving structure tests as force/displacement curves we could measure displacements of lattice points and estimate the summed loading between them, then compute macromaterial properties which properly reflect the performance of the parent material within the form of that particular structure. The importance of E = stress/strain is the universality of the materials like steel whose E we are constantly using: they are common to all manner of structures. And so it is with form: building frames, tubular steel lattices and prestressed concrete are examples of macromaterials that are used with minor variations over and over again. Their equivalent macromaterial properties like density, Young modulus, strength, toughness, durability and cost can be expected both to give a direct insight to their performance and to be usable directly in design, simplifying the process.

Figure 10 shows two four-chorded GRP lattices 1 m. tall buckling under axial compression, when were tested in 1979. Both had identical chords, but one was singly cross-braced and the other doubly so. The force/displacement curves for both structures are shown in the middle of the figure. Notice that doubling the bracing doubled the buckling load, but their failure modes were different. In the first the bracing did not buckle, only the chords, while in the second the bracing buckled as well, giving a much sharper peak to the load/displacement curve, a more brittle response. The chords had their centres on a rectangle 230 mm by 150 mm, so the radii of gyration of their sections were 115 mm and 75 mm. The ellipse with these radii of gyration has diameters 460 mm and 300 mm and an area of 108,000 mm^2. Dividing axial force by this area and dividing displacements by 1 m. transforms the force/displacement curves into macromaterial stress/strain curves.

The mass of the specimens is estimated as 650 gm and 800 gm. Dividing these masses by the volume described by the equivalent section multiplied by specimen length gives macromaterial densities of 6 and 7.5 kg/m^3, less than one thousandth that of steel. Such low densities are nothing new: the 51,800 tonnes of steelwork in the Forth Railway Bridge is spread within an envelope whose volume is 1,820,000 m^3, having a mean density of 28 kg/m^3. The strengths of the two macromaterials of 0.070 MPa and 0.128 MPa should be compared with about one thousandth of the strength of steel if we are to compare materials of the same density. For prismatic forms, like I- and box-section having a sparsity $i = A'/A$, the best we can expect from a parent material having density ρ, Young modulus E, and strength σ, is a macromaterial having density ρ/i, Young modulus E/i, and strength σ/i. When properties are so finely and uniformly reducible as they are in fluids we call it dilution, but this optimum solid material dilution is only achievable for prismatic members in their axial directions. Due to local buckling the strengths of macromaterials can be excessively diminished compared to their stiffness and density, and due to lateral members like bracing, stiffness can diminish more than the density.

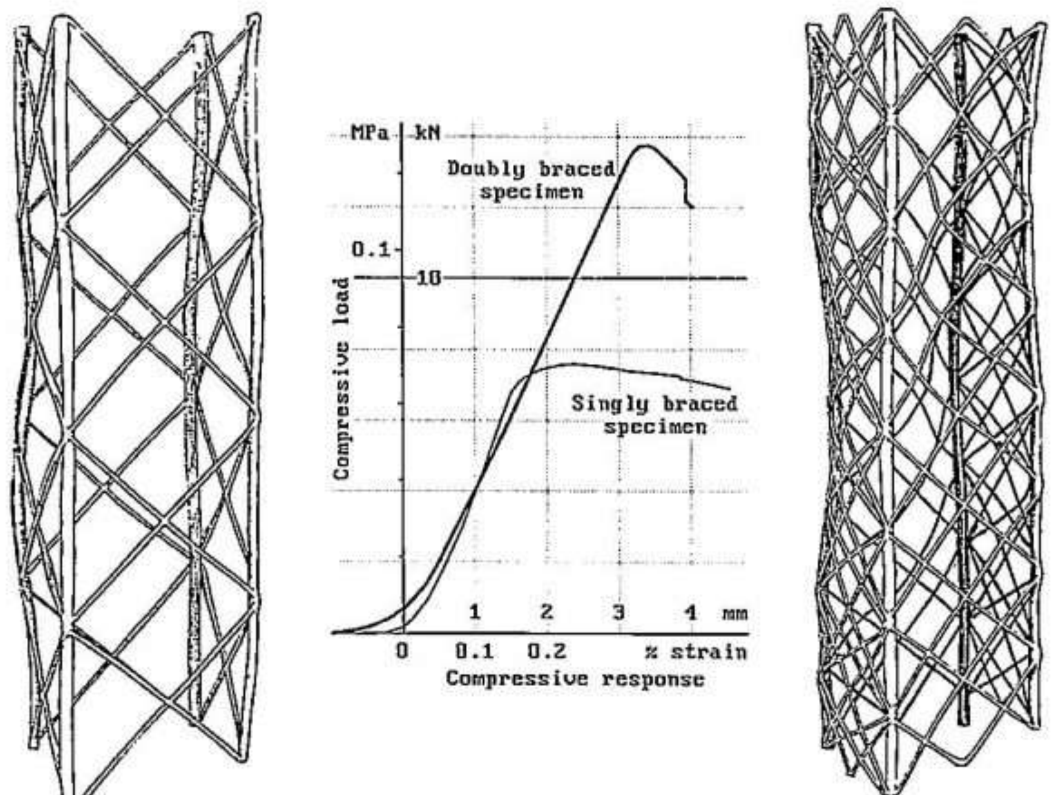

Figure 10. Two GRP lattices each 1 m. tall were tested in compression and are sketched in their buckled states. Their force/displacement curves are shown in the middle. The curve with the thicker line refers to the right hand doubly braced lattice. Doubling the bracing doubles the failure load but alters the failure mode from chord buckling to brace buckling and results in brittler behaviour at failure as indicated by the sharper load shedding of the thicker curve.

For some quantification of these effects refer to[2].

If principal properties like density, stiffness and strength can be transformed by such simple rules, what about toughness? Figure 10 shows that differences in bracing can radically alter the shape of a stress/strain curve. Macromaterials can show non-linear 'yielding' type responses that are nothing to do with ductile flow, and they can fail in true compression. In these respects they present engineering researchers with unexplored territory. Of particular interest is how important a yielding response of a macromaterial is to structural integrity: we know that a yielding response of a ductile parent material is essential in tensile applications, and that brittle parent materials like GRP can provide a yielding response in compression, as in Figure 10, or in the response of elastic Euler strut buckling. We also know that combined buckling and parent material plasticity can produce the

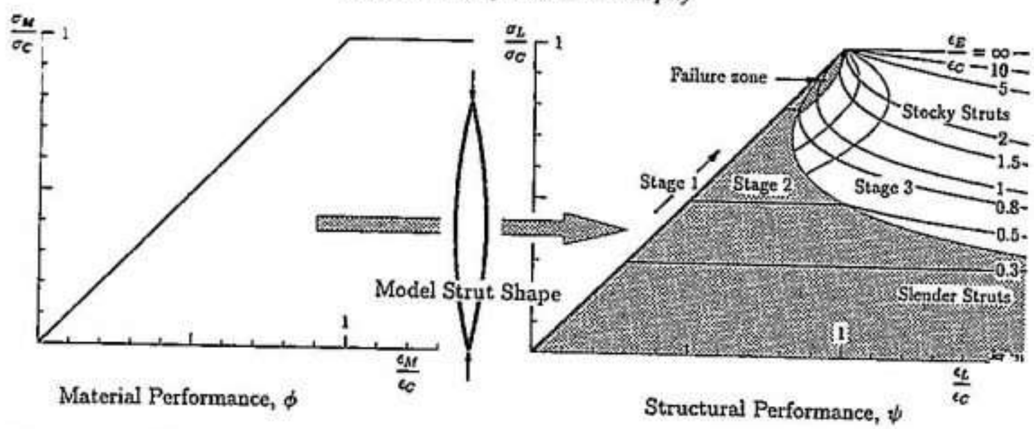

Figure 11. The elastoplastic material on the left is 'fed' into a *model structure*, the lenticular two-flanged structure in the middle of the figure. The response to compressive loading is shown on the right, each curve for a different slenderness.

most brittle responses, as shown in Figure 11, taken from[3]. The optimum strut, which reaches plasticity and elastic buckling at the same load has the most brittle imaginable response curve, like a breaking wave, the one marked '1' on the right of the figure. It seems clear that the shape of the stress/strain curve can be transformed both ways in compression, from brittle to yielding or from yielding to brittle, depending on the shape and form of the macrostructure. Certainly increased strength can be at the expense of increased brittleness, and indications are that very strong brittle parent materials are better for macromaterials working in compression than weaker ductile ones. This phenomenon needs further investigation as it may shed new light on the principles by which we should design.

THE WAY FORWARD

Presently material concepts for engineers are mostly confined to a laboratory scale, the scale defined by the width of our material test specimens. Materials' performance at these scales is widely available, and the materials industry is thriving on the development and manufacture of the burgeoning number of new materials coming to the market place. In contrast engineers have great difficulty knowing how to form new materials appropriately. We suggest this is because of a need for more useful shape concepts. The macromaterial concept that has been described is offered to meet this need. Ashby is recognising this need and in his recently published book[4] he not just provides a wealth of materials data in accessible form but introduces shape factors that for prismatic members effectively generate macromaterials.

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