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Damage Accumulation in Structures

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ABSTRACT

Four types of defect are described, and the hierarchical nature of shape, form and material is explored. Defects and failure are also shown to be hierarchical, and a structure's damage is defined as the sum of its defects. The concept of material toughness is linked to structural redundancy. Damage is described in two ways: geometrically and materially, the latter describing performance loss quantitatively. The paper concludes with a simple numerical example of how damage affects reliability.

1 INTRODUCTION

An inevitable consequence of working in a competitive world is that our structures and machines should be designed to perform to their maximum capability. Skilful designing is needed in this situation to be able to minimize the overall costs, which usually include the costs of operation, maintenance and repair as well as those of manufacture. Reducing the costs of manufacture too much may result in a structure that is too weak to perform reliably in service. The consequential damage and expense of service failures should be considered during design.

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Structures can become damaged in service by normal operation, for example by fatigue and creep, and by external events such as local impact. Accumulation of damage during service by either means is gradually going to reduce their reliability. This paper discusses damage and how it affects reliability.

Current codes of practice used for the design of structures and pressure vessels are deterministic, using the permissible stress approach. A structure that satisfies a code is deemed to be safe; 'How safe?' is not a question these codes have been written to answer. The current trend is towards incorporating limit state philosophy into the deterministic framework, and introducing partial safety factors on loads, load combinations and material strength and others to allow for uncertainties in assessing the effects of loading, for weaknesses in the structure due to deficiencies other than in material strength and for the consequences of failure.[1] During preparation, such codes can be calibrated using the so-called Level 2 techniques, to ensure similarity of reliability across the range of different components capable of satisfying a code. How this has been done for BS 5400[2] is described by Thoft-Christensen and Baker.[3]

Currently there are three documents which give guidance on acceptance levels for crack-like defects: ASME XI,[4] PD 6493,[5] and the CEGB's 'R6' method.[6] These references present methods for determining crack stability using the principles of fracture mechanics, and are used extensively in the nuclear power and offshore industries to determine defect acceptance levels during fabrication, and to establish whether or not cracks detected in service need to be repaired.

What follows is exploratory; some of the ideas are novel and some of the terminology is new.

2 DEFECTS

The physical world abounds with structural defects, at many levels. At the crystalline level we have the *dislocation*, the *void* and the *crack*. Associated with the void is the inclusion. *Deviation* is being used in this paper as the name for any shape imperfection; such a defect is likely to occur on a larger scale than that of the crystal lattice, in structures and anisotropic materials. These are four principal types of defect. The geometry of each is illustrated in Fig. 1.

As might be expected, each defect generally has a harmful effect upon the performance of the host material. Typical effects upon the host material's stress/strain curve are shown in Fig. 2. Dislocations bring loss of strength with an apparent benefit of great deformability. Inclusions, voids and cracks all bring loss of strength by themselves, but in the presence of

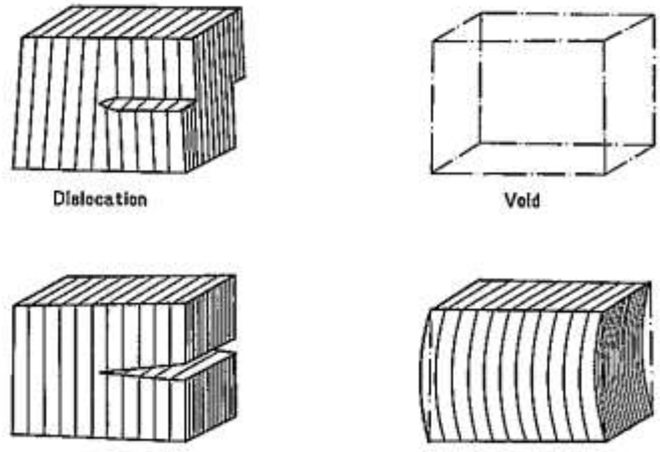

Fig. 1. The geometry of four principal types of material defect.

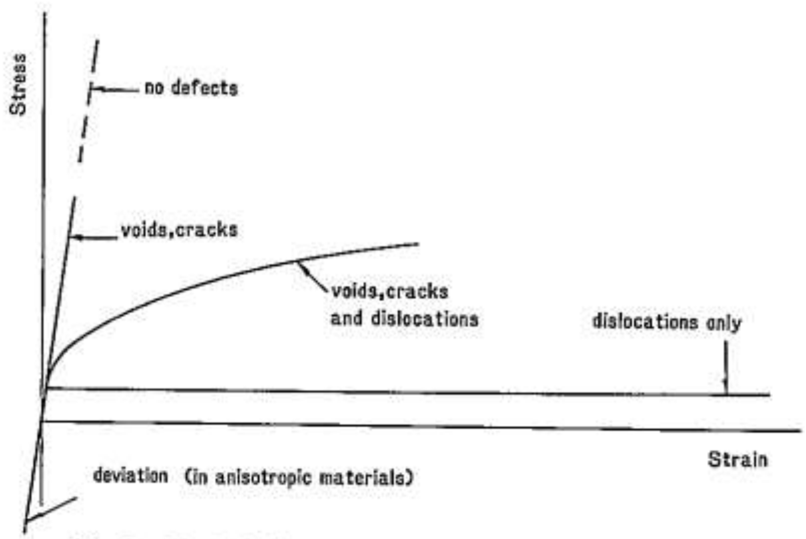

Fig. 2. Typical effects of defects upon material performance.

dislocations, inclusions may provide increased strength by obstructing the dislocations' movement. Cracks of sufficient size will render a ductile material brittle as well as reducing its strength. Deviation is recognizable as out-of-trueness in structures and reduces compressive strengths.

3 SHAPE, FORM AND MATERIAL

In Section 2, two separate aspects of each defect were considered: its geometry and its effect on material, the latter being expressed in terms of material performance. The relationship between geometry and material has been discussed by Parkhouse,[7] who demonstrated that structuring is a process of material dilution, and that the structuring process is hierarchical. This section of the present paper describes the relationship between geometry and material in simple qualitative terms.

A shape is entirely geometric. It possesses points, lines, surfaces and a volume, and absolutely nothing else. A material may be considered as possessing any number of properties, like colour, density, stiffness, strength, stress and temperature, but absolutely no geometric property except that it is an occupier of volume. Shape and material are thus complementary, and are linked by the fact that material can occupy a shape's volume to make a structure. This relationship may be expressed by the equation:

$$\text{Shape} \times \text{material} = \text{a structure} \qquad (1)$$

which is illustrated in the top half of Fig. 3. In this figure, geometry and material are treated two-dimensionally, so that shape possesses points, lines and an area, and material is an occupier of area. Material is then analogous to ink and is represented by the grey shading. Notice that it is impossible to represent ink on paper without giving it a shape, so the irregular outline used has been chosen for its inconspicuousness, and it is not to be considered as belonging to the material.

A form is entirely geometric. It is like a shape except that it repeats itself indefinitely in at least one direction. The chain-dotted line in the lower half of Fig. 3 is a form which repeats itself in two directions. All forms are

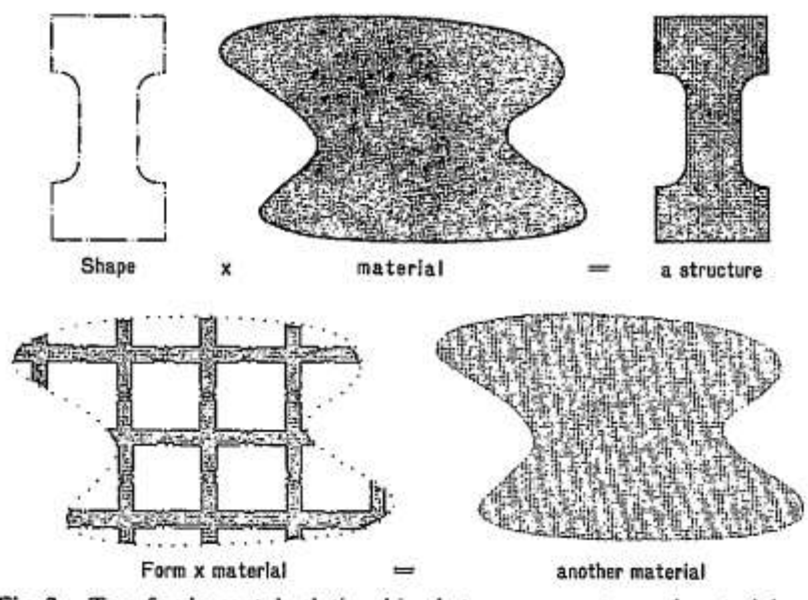

Fig. 3. Two fundamental relationships between geometry and material.

unbounded and therefore are as difficult to represent on paper as was material in the upper half of the figure. Therefore the same irregular outline has been used around the form, but again it is not to be considered as belonging to the form. Form is as complementary to material as shape, and linking form to material produces a repeating structure which is unbounded, just like a material. It has been shown in ref. 7 that properties of another material can be derived from those of the original material using the form's geometry. This relationship may be expressed by the equation:

$$\text{Form} \times \text{material} = \text{another material} \tag{2}$$

which expresses the concept of the structuring of material. Form may be considered to be a material transformation. The transformed material, on the right of the equation, is less dense, less stiff and less strong than the original, because the original material is mixed with the space outside the form. This is why structuring is a process of material dilution. It has been suggested that the transformed materials should be called 'virtual materials'. Although they fully satisfy the material definition given earlier in this section, it would be confusing not to distinguish them from real materials.

There are families of forms. One such family is the cylindrical tube, its members ranging from thick-walled tubes to thin-walled shells. Another such family is the lattice; this is a very complicated family embracing a multitude of different geometries as well as different member slendernesses. Simple examples of material transformations for these forms are given in ref. 7.

Since every material can in principle be formed into every form, the set of conceivable virtual materials is even more densely populated than the set of forms; the more so since eqn (2) is recursive, so that virtual materials derived from the right-hand side may then be used in the left-hand side to be transformed into another virtual material. A lattice of tubes is an example of form combination.

This form combination is the hierarchical element in structuring. It is illustrated in Fig. 4 by three models representing the same tower which is structured in two stages by the same 'ladder' form. This form repeats in one direction, and is compatible with eqn (2). Figure 4 shows the width of the virtual solid elements representing the lengths of laddered material as somewhat greater than the width of the ladder; the reason for this is explained in ref. 7. Each level is considered in the light of eqn (1). The Level 0 structure belongs to the lowest level, whose material is the real black material and whose shape is the surface of the real material, the most intricate surface. The Level 1 structure belongs to the next level up, whose material is virtual and dark grey. Its surface is the Level 1 shape. The Level

2 structure belongs to the highest level whose material is again virtual and is portrayed as light grey. Its surface is totally convex; evidence that it belongs to the highest level.

The structure belonging to Level 0 is the real structure, while those belonging to the higher levels are virtual structures. 'Real' and 'virtual' are only used to clarify where the concept of material is being extended beyond orthodox usage; in theory both types of material are as real as each other. Virtual materials can be expected to be relevant to structural performance and to be useful when more information on virtual material properties becomes available.

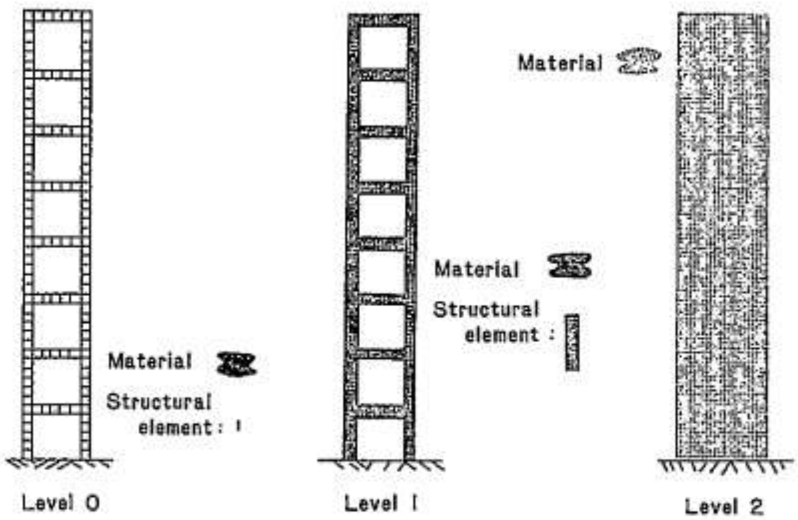

Fig. 4. Three hierarchical levels of a tower.

It might be asked whether the simpler, higher-level models can possibly model the complexity evident in the real structure. This is like wondering whether the material properties of a steel are adequate for describing the effect of all the different grain shapes, inclusions, the configuration of trace element atoms within the crystal lattice, and the many other low-level details. Just as conventional material properties do adequately describe the performance of extremely complex systems, so each material transformation can take proper account of the effects of the local instabilities and the local defect distribution, so that the higher-level virtual material properties will reflect their presence. This could prove to be an effective way of handling the complexity of multi-level structures. The interaction of local buckling modes in not easily analysed applying conventional Level 0 models.

4 DAMAGE

Defects are also hierarchical, and so is failure. A layer of paint may have local weaknesses. Eventual local failure of this paint layer would initiate the development of a corroding pit in the underlying steel. In this example there are two levels, and failure at the lower level results in a defect at the higher one. This is generally true of hierarchical deterioration. Another example is of a fatigue crack at the toe of a weld. This fatigue crack is a defect in the platform made with latticed tubes, the Level 0 structure. When it breaks through the wall there is a Level 0 local failure, and a sudden increase in the defectiveness of the Level 1 structure, the platform made with a lattice. The crack then grows around the circumference of the tube, until the tube fails. This is a Level 1 local failure and a sudden increase in defectiveness in the Level 2 structure, the platform. The final stage might be the progressive failure of the adjacent tubular members in the lattice, which would be more highly stressed following the first failure. This is equivalent to a crack growing within the highest-level virtual material.

Defining Level 0 as real material is only a convenience. Even real materials are structured; metals have granular forms, and grains have crystalline forms of atoms. When the smallest of cracks grows, atomic bonds are broken, which is local failure on a very low level. So it is probably true that every increment of defectiveness is associated with local failure at some level. This is an argument for defects being irreversible, because failure is irreversible, and a growing defectiveness must be leading to failure at higher and higher levels. The ultimate failure is failure at the highest

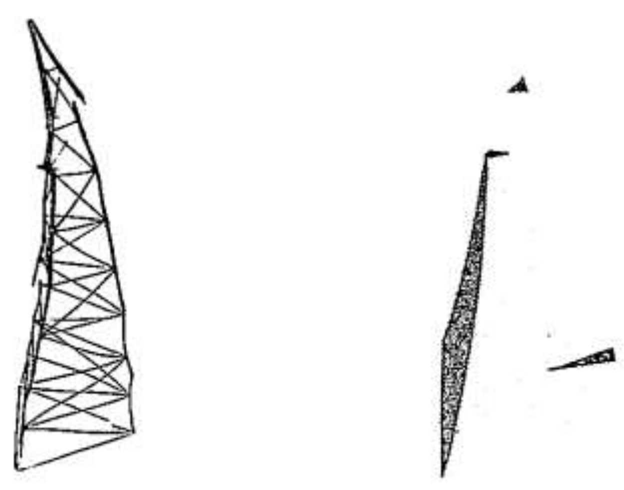

Fig. 5. An example of extreme structural damage.

level, though a structure may reach a serviceability 'limit state' before this happens. Such limit states are likely to coincide with damage criteria.

At any stage in the life of a structure its damage is the sum of its defects. By this definition, even new structures are damaged to some extent since they all possess defects. Defects may be considered to be the seeds of failure, producing damage as they grow.

Figure 5(a) shows a photo of a tower, 450 mm tall, made of spaghetti, severely damaged by exposure to a damp atmosphere. The Level 0 material is pasta and Level 1 is the highest level. Many Level 0 elements are bent (deviation of Fig. 1) and many are broken. These breaks are equivalent to cracks in the material of the Level 1 model, illustrated in Fig. 5(b). The Level 1 structure is also bent.

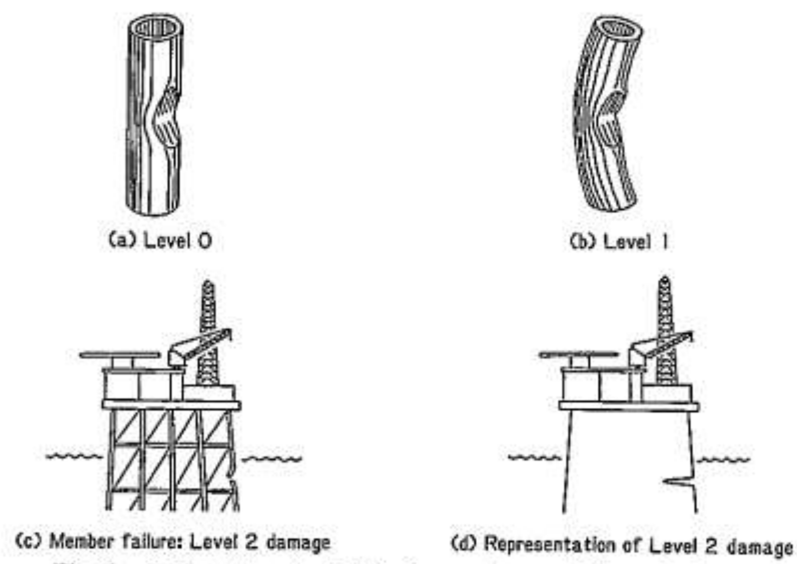

Fig. 6. Different levels of defectiveness in an offshore platform.

Figure 6 illustrates an offshore platform with local wall and member deviations in a tube, local defects at Levels 0 and 1. Member failure is a Level 1 failure and may be considered to introduce a crack into the Level 2 structure, as illustrated at the bottom of the figure.

5 REDUNDANCY

A system is called a redundant system when not all of its members are needed for it to be able to function. It can suffer member failure without itself failing; but the damage to the system results in a reduction of its redundancy.

The redundancy of a system depends on its loading, as well as upon its geometric configuration; for example, a cracked bar becomes less redundant in the vicinity of the crack tip as the tensile load in the bar is increased. At the fracture load, the bar loses all its redundancy and the failure of one atomic bond will cause the whole bar to fail. This brittle behaviour is a result of a loss of redundancy at a lower level. Conversely, ductility is associated with high redundancy, which is why ductile materials are tolerant to defects and to abuse. This property is called toughness.

The conventional view of structural redundancy does not embrace toughness. It confines its domain to the unloaded Level 0 structure. This restricted view assesses a bar as statically determinate, not redundant, without regard to cracks or the toughness of its material, since both these attributes are related to redundancy at levels lower than Level 0.

The concept of material toughness could be extended to describe the benefits of structural redundancy by extending its domain to include virtual materials. It seems probable that toughness is most necessary for reliable performance at the lowest level, becoming progressively less necessary at higher levels. A reason for this is the effect of scale; lowest-level elements are so small that each is very vulnerable to the many everyday events in its vicinity. If Level 0 material were very lacking in toughness, the mere brush by a feather could precipitate the failure of a large member supporting a large load. At the other end of the scale there is always a highest-level structure which is statically determinate, for example the highest level structures in Figs 4, 5 and 6.

6 DESCRIBING DAMAGE

As with defects, there are two aspects of damage: its geometry and its harmful effect upon performance. Both can be described and measured. Geometrically, damage is a collection of defects, and may be described in terms of defect types, sizes and their distributions. Materially, damage is a reduction in material performance described in terms of properties like stress/strain relationships and fracture toughness. The reduced performance can be derived in principle from a knowledge of the damaged geometry in accordance with eqn (2), if this relationship is considered to represent a mathematical material transformation carried out on the relevant material performance parameters. How this should be done is currently a matter for research.

It is interesting to consider how the four types of defect of Fig. 1 appear as damage at different levels of a structure's hierarchy. The dislocation is essentially a lack of fit. At the atomic level of metals it has a special significance responsible for plasticity at Level 0. At higher levels, even in metals, the dislocation has no mobility and results in residual stress. Residual stresses due to welding are mainly Level 0 damage, though a seam weld along a tube may be expected to introduce a residual stress field extending right round the tube's circumference. This is Level 1 damage. Forces applied in a fabrication yard to ensure closure at a joint of a tubular frame before welding would result in residual stresses around at least one cell of the framework, and these would be Level 2 damage.

The void or inclusion appears at Level 0 as porosity in weld metal or concrete, and as chemical or physical transformation, such as corrosion in steel or alkaline aggregate reaction in concrete. At higher levels, voiding is associated with holes, openings and extensive member failure, as might be caused by an explosion. Cracks and deviations have already been shown to be hierarchical. All four types of defect are likely to exist at many levels in all structures. In principle, the distribution within a structure of each defect type at each level could be described statistically. Both the average defectiveness and the extreme defectiveness at each level are of importance. The locations of the most extreme defects in each level are where local failure is most likely to occur.

7 THE EFFECT OF DAMAGE ON RELIABILITY

Figure 7 shows two structural elements, a cracked one subjected to a random tensile load, and a bent one subjected to a random compressive load. In both cases the loading is sufficiently intense for the single top level defect to increase with time; the crack grows by a fatigue process near the crack tip which is sensitive to the fluctuating component of loading, and the amplitude of the bend increases by a creep process, the rate of which is sensitive to the mean component of loading. The peak load that each could sustain reduces as their damage increases, and if the characteristics of the random loading stay constant, then the reliability of each component is progressively reducing. Reliability is expressed as the probability of surviving a given period of time.

To illustrate the effect of damage on reliability, consider the large steel plate shown in Fig. 8. It is 1000 mm wide and has a weld defect, initially 5 mm deep, at one end. The steel has a yield strength of 250 MPa and a fracture toughness of 150 MPa $\sqrt{\text{m}}$. The loading is such that the initial crack growth rate is 0·2 mm/year and the peak annual loading produces a uniform tensile stress in the uncracked steel plate of 150 MPa, with a standard deviation of 30 MPa. This annual peak stress is a normally distributed random variable.

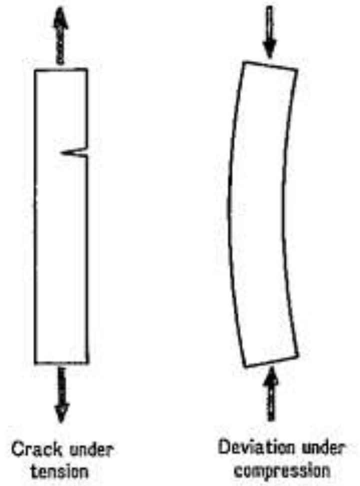

Fig. 7. Sensitive combinations.

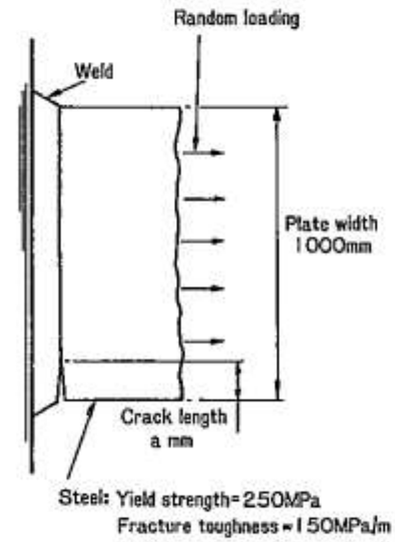

Fig. 8. Subject of damage calculations.

The response of this structural element to this loading is based on the following assumptions:

(a) the strength of the element, U, is related to a notional plastic strength, P (based on the yield strength acting on the uncracked area), and a notional fracture strength, F (derived by linear elastic fracture mechanics), by the equation:

$$1/U^2 = 1/P^2 + 1/F^2 \tag{3}$$

(b) The crack growth rate is assumed to increase in proportion to the crack length raised to the power of 3/2.

The outcome of these assumptions is shown in Fig. 9. The geometric description of damage is the crack length, illustrated by the lower curve, and the material description of damage is the loss of strength, illustrated by the amount the upper curve falls beneath the 250 MPa undamaged strength line. Both descriptions increase and accelerate with time, but the material description is the significant one for assessing safety since it may be compared directly with the applied loading of (150, 30) MPa.

The instantaneous probability of failure is defined as $\Delta P/\Delta T$ where ΔP is the probability of failure in a very short interval ΔT. It can be derived from the strength and the loading, and is shown in Fig. 10. The cumulative probability of failure is based on an integral of the instantaneous probability of survival, and is the probability of failure before a given time. It is the upper curve in Fig. 10. It shows that the most likely life of the element is about 39 years. However, if continued service were conditional

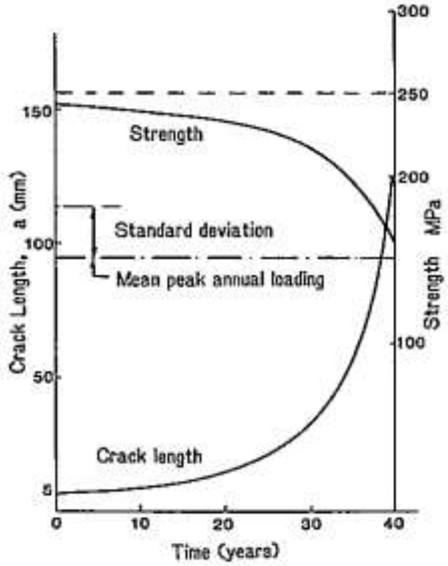

Fig. 9. Damage history.

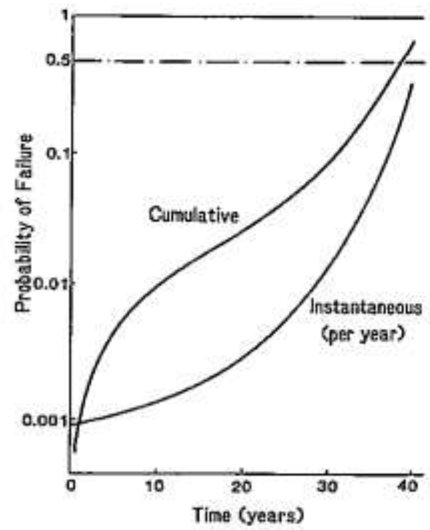

Fig. 10. Reliability history.

on sufficient reliability, then typical safe lives might be 4 years and 29 years, corresponding to annual failure probabilities of 1/1000 and 1/100.

Figure 10 has demonstrated how, for a simple example, damage affects reliability. The damage is a crack being grown by fatigue loading, but the alternative, illustrated in Fig. 7, could have been used as an example of creep buckling; if it had been, similar curves would have been produced to those of Figs 9 and 10.

8 CONCLUSION

Damage should always be considered in structural reliability analysis.

The hierarchical nature of structures enables their complexity to be considered level by level. This approach potentially reduces the reliability analysis of any structure to separate analyses of its structural forms, followed by an analysis of the statically determinate structure representing the highest hierarchical level. Taking a hierarchical view of a severely damaged offshore platform, as shown in Fig. 6(d), its analysis is potentially very similar to that of a simple plate, illustrated by Figs 8, 9 and 10.

9 EPILOGUE

St Paul[8] has referred to the creation as being in bondage to decay. The reader may have recognized in this paper a description of a mechanism of decay of both inanimate matter and of order. Two features of the mechanism, the irreversibility of its processes and the continual reduction of the total order in a system, are in harmony with the Second Law of Thermodynamics.

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